ΦΥΣΙΚΗ ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΥ 2019 (ΓΕΛ)

ΘΕΜΑ Α

Α1) β

Α2) γ

Α3) α

Α4) γ)

Α5) α) Λ
β) Σ
γ) Λ
δ) Σ
ε) Σ

ΘΕΜΑ Β

Β1) Σωστό το β)

Α.Δ.Ο $\vec{P}_{αρχ} = \vec{P}_{τελ} \Rightarrow$

$m_1 \cdot U_S = (m_1 + m_2) \cdot U_Σ$

$m_1 \cdot U_S = 2m_1 \cdot U_Σ$

$U_Σ = \frac{U_S}{2}$

$$f_1 = \frac{u_{ηχ} + 0}{u_{ηχ} + U_S} \cdot f_S$$

$$f_2 = \frac{u_{ηχ}}{u_{ηχ} + \frac{U_S}{2}} \cdot f_S$$

$$\frac{f_1}{f_2} = \frac{u_{ηχ} + \frac{U_S}{2}}{u_{ηχ} + U_S} \Rightarrow$$

$$\frac{f_1}{f_2} = \frac{u_{ηχ} + \frac{u_{ηχ}}{40}}{u_{ηχ} + \frac{u_{ηχ}}{20}} = \left(\frac{41 \cdot u_{ηχ}}{40} \cdot \frac{20}{21\, u_{ηχ}} \right) = \frac{41}{42}$$

$$\Rightarrow \boxed{\frac{f_1}{f_2} = \frac{41}{42}}$$

Β2) Σωστό το γ)

Στάθμη νερού $\Rightarrow$ $Π_1 = Π_2 \Rightarrow A_2 \cdot U_1 = A_3 \cdot U_2$

$A_1 = 2A_2 \Rightarrow A_2 = \frac{A_1}{2}$

$A_3 = \frac{A_2}{2}$

$\Rightarrow A_2 \cdot U_1 = \frac{A_2}{2} \cdot U_2$

$\Rightarrow \boxed{U_2 = 2 \cdot U_1}$ (1)

$\Rightarrow \boxed{U_3 = 2 \cdot U_2}$

$Π_1 = Π_2 \Rightarrow A_1 U_1 = A_2 U_2 \Rightarrow$

$2A_2 \cdot U_1 = A_2 U_2 \Rightarrow \boxed{U_2 = 2 \cdot U_1}$ (2)

$\hookrightarrow U_1 = \frac{U_2}{2}$

Bernoulli (Δ) → (Γ) (h=0 στο Γ)

$$P_1 + \frac{1}{2}\rho U_1^2 + 0 = P_2 + \frac{1}{2}\rho U_2^2 + 0 \Rightarrow$$

$$P_1 - P_2 = \frac{1}{2}\rho (U_2^2 - U_1^2) \quad (2) \Rightarrow$$

$$P_1 - P_2 = \frac{1}{2}\rho (4U_1^2 - U_1^2) \Rightarrow$$

$$P_1 - P_2 = \frac{1}{2}\rho \cdot 3U_1^2 \xrightarrow{P_2 = Patm} P_1 - Patm = \frac{3}{2}\rho U_1^2 \quad (3)$$

Στην ελεύθερη επιφάνεια νερού

$$P_1 = Patm + \rho g h_1 \Rightarrow P_1 - Patm = \rho g h \quad (4)$$

$$(3)(4) \Rightarrow \frac{3}{2}\rho U_1^2 = \rho g h \Rightarrow h = \frac{3U_1^2}{2g}$$

Δοχείο νερού

Bernoulli (Ε) → (Ζ) (h=0 στο Ζ)

$$P_E + \frac{1}{2}\rho U_E^2 + \rho g H = P_Z + \frac{1}{2}\rho U_Z^2 + 0 \Rightarrow$$

$$Patm + 0 + \rho g H = Patm + \frac{1}{2}\rho U_3^2 \Rightarrow$$

$$\rho g H = \frac{1}{2}\rho U_3^2 \Rightarrow$$

$$H = \frac{U_3^2}{2g}$$

Άρα $$\frac{h}{H} = \frac{\frac{3U_1^2}{2g}}{\frac{U_3^2}{2g}} = \frac{3U_1^2}{U_3^2} = \frac{3\left(\frac{U_2}{2}\right)^2}{(2U_2)^2} = \frac{3\frac{U_2^2}{4}}{4U_2^2} = \frac{3U_2^2}{16U_2^2}$$

$$\boxed{\frac{h}{H} = \frac{3}{16}}$$

Β3) Σωστό το

ΘΜΚΕ (Α)→(Δ)

$$\frac{1}{2} I_p \cdot \omega^2 - 0 = W_F \Rightarrow$$

$$\frac{1}{2} I_p \omega^2 = F \cdot L \cdot \frac{\pi}{2}$$

$$\frac{1}{2} \cdot \frac{1}{3} ML^2 \omega^2 = F \cdot L \cdot \frac{\pi}{2}$$

$$\omega^2 = \frac{3F \cdot L \cdot \pi}{ML^2}$$

$$\omega^2 = \frac{3 \cdot 9\pi \cdot 1 \cdot \pi}{3 \cdot 1^2}$$

$$\omega^2 = 9\pi^2$$

$$\boxed{\omega = 3\pi \ rad/s}$$

Ιδιαίτερη → $\vec{L}_{αρχ} = \vec{L}_{τελ}$

$$I_p \cdot \omega = I_{ολ} \cdot \omega_\Sigma$$

$$\frac{1}{3} ML^2 \cdot \omega = \left(\frac{1}{3} ML^2 + mL^2\right) \cdot \omega_\Sigma$$

$$\frac{1}{3} \cdot 3 \cdot 1^2 \cdot 3\pi = \left(\frac{1}{3} \cdot 3 \cdot 1^2 + 1 \cdot 1^2\right) \omega_\Sigma$$

$$3\pi = (1+1)\omega_\Sigma$$

$$\boxed{\omega_\Sigma = \frac{3\pi}{2} \ rad/s}$$

Αφού $F = 0 \Rightarrow \omega_\Sigma = σταθερή$

Άρα $\omega_\Sigma = \frac{\Delta\theta}{\Delta t} \Rightarrow$

$$\Delta t = \frac{\Delta\theta}{\omega_\Sigma} = \frac{\frac{\pi}{2}}{\frac{3\pi}{2}}$$

$$\Delta t = \frac{2\pi}{3 \cdot 2\pi} \Rightarrow$$

$$\boxed{\Delta t = \frac{1}{3} \ sec}$$

ΘΕΜΑ Γ

Γ1) $\Sigma F_y = 0$ (Θ.Ι)

$W = F_{ελ} \Rightarrow$

$m_1 g = K \Delta l \Rightarrow$

$10 = K \cdot 0{,}05 \Rightarrow$

$K = \frac{10}{0{,}05} = \frac{1000}{5}$

$\boxed{K = 200\ N/m}$

Κρούση $\Rightarrow$ ΑΔΟ $\Rightarrow$ $\vec{P}_{αρχ} = \vec{P}_{τελ}$

$\Rightarrow m_1 \cdot v_1 + m_2 \cdot v_0 = m_\Sigma \cdot v_\Sigma$ ($v_1 = 0$)

$\Rightarrow 1 \cdot v_0 = 2 \cdot v_\Sigma$

$\Rightarrow v_\Sigma = \frac{v_0}{2}$ (1)

(Ν.Θ.Ι) $\Sigma F_y = 0 \Rightarrow$

$(m_1 + m_2) g = K \cdot \Delta l' \Rightarrow$

$20 = 200 \Delta l' \Rightarrow$

$\Delta l' = \frac{20}{200} \Rightarrow \underline{\Delta l' = 0{,}1\,m}$

Γ2)

Στην $t = 0$ ισχύει $x_1 = \Delta l' - \Delta l$

$x_1 = 0{,}1 - 0{,}05$

$\underline{x_1 = 0{,}05\,m}$

από ΝΘΙ.

Όπως φαίνεται μέχρι το ΦΜΕ ($U = 0$). Άρα είναι η αρχική και θέση όπλου $\boxed{A = \Delta l' = 0{,}1\,m}$

<u>ΑΔΕΤ (για ανεξάρτητα)</u>

$E = K + U \Rightarrow$

$\frac{1}{2} K \cdot A^2 = \frac{1}{2}(m_1 + m_2) \cdot v_\Sigma^2 + \frac{1}{2} \cdot K \cdot x_1^2$

$200 \cdot 0{,}1^2 = 2 \cdot v_\Sigma^2 + 200 \cdot 0{,}05^2$

$200 \cdot 0{,}01 = 2 \cdot v_\Sigma^2 + \frac{200}{4} \cdot \frac{1}{2}$

$2 = 2 \cdot v_\Sigma^2 + \frac{1}{2} \Rightarrow$

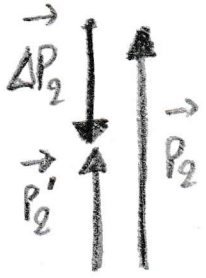
$2 v_\Sigma^2 = 1{,}5 \Rightarrow$

$v_\Sigma = \sqrt{\frac{1{,}5}{2}} = \sqrt{0{,}75} = 0{,}5\sqrt{3}\ m/s$

(1) $\Rightarrow v_0 = 2 v_\Sigma = 2\sqrt{\frac{1{,}5}{2}} = 2 \cdot 0{,}5\sqrt{3} = \sqrt{3}\ m/s$

Άρα $K_2 = \frac{1}{2} m_2 v_0^2 = \frac{1}{2} \cdot 1 \cdot (\sqrt{3})^2 = \boxed{1{,}5\,J}$

Γ3) $\Delta\vec{P}_2 = \vec{P}_2' - \vec{P}_2 \Rightarrow$ (+↑)



$\Delta P_2 = m_2 U_2' - m_2 U_0$

$\Delta P_2 = 1 \cdot 0{,}5\sqrt{3} - 1 \cdot \sqrt{3}$

$\Delta P_2 = -0{,}5\sqrt{3}$ Kgm/s

$\Rightarrow \boxed{|\Delta P_2| = 0{,}5\sqrt{3} \text{ Kgm/s}}$

με φορά προς τα κάτω (αντίθετα από θετική φορά) τα λάνκων

Γ4) $x = A\eta\mu(\omega t + \phi_0)$

$x = 0{,}1\,\eta\mu(\omega t + \phi_0)$

$\Rightarrow \boxed{x = 0{,}1\,\eta\mu(10t + \frac{\pi}{6})\ (SI)}$

- $D = K = (m_1 + m_2)\omega^2$

$\Rightarrow \omega = \sqrt{\frac{K}{m_1+m_2}} = \sqrt{\frac{200}{2}}$

$\Rightarrow \underline{\omega = 10 \text{ r/s}}$

- <u>για το ϕ_0</u>

$x = A\eta\mu(\omega t + \phi_0)$
$t = 0$
$x = x_1 = +0{,}05$
$U_\Sigma > 0$
$\Rightarrow$

$0{,}05 = 0{,}1\,\eta\mu\phi_0 \Rightarrow$

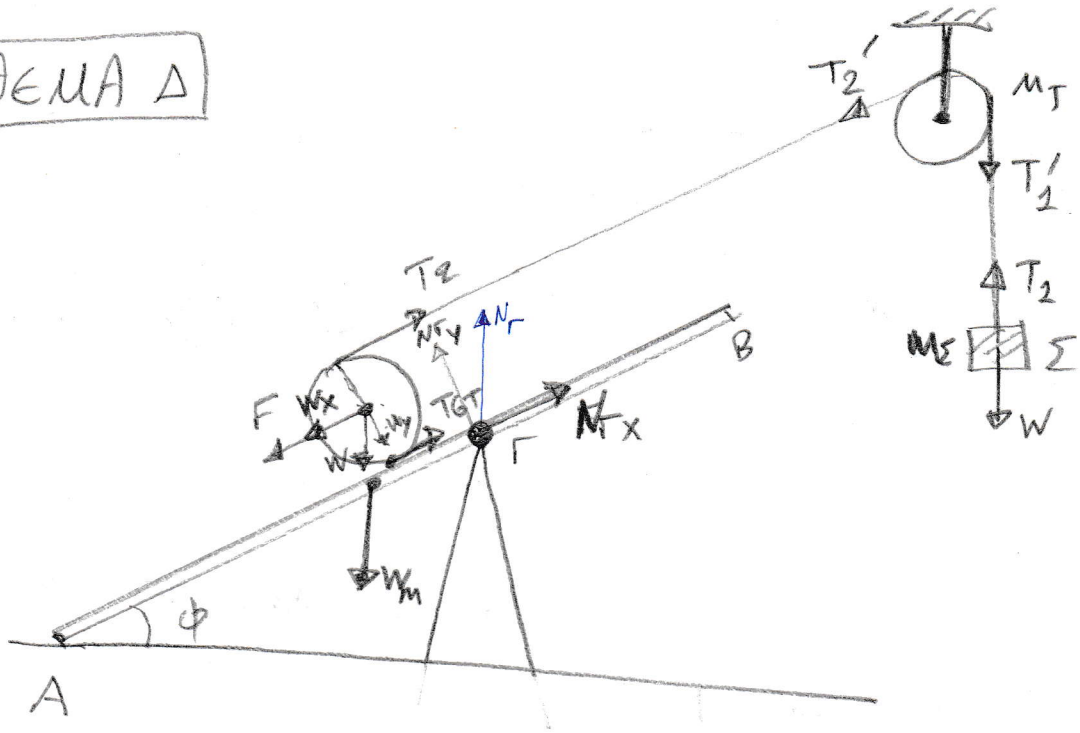
$\eta\mu\phi_0 = \frac{1}{2} = \eta\mu\frac{\pi}{6}$

$\phi_0 = 2k\pi + \frac{\pi}{6}$ ή $\phi_0 = 2k\pi + \pi - \frac{\pi}{6}$

$U_\Sigma = 0 \Rightarrow \boxed{\phi_0 = \frac{\pi}{6} \text{ rad}}$ ή $\phi_0 = \frac{5\pi}{6}$ rad

δεκτή αφού $\sigma\upsilon\nu\frac{\pi}{6} > 0 \Rightarrow U > 0$

ΘΕΜΑ Δ



Τα νήματα αβαρή + ιδανικά

Δ1) Ισορροπία Σ

$\Sigma F_y = 0 \Rightarrow$

$M_\Sigma g = T_2 \Rightarrow$

$T_1 = 20\,N$

Ισορροπία τροχαλίας

$\Sigma \tau_{(K)} = 0 \Rightarrow$

$T_1 \cdot R_T - T_2 \cdot R_T = 0 \Rightarrow$

$T_1 = T_2 \Rightarrow$

$T_2 = 20\,N$

$T_1 = T_1'$

$T_2 = T_2'$

Ισορροπία ψωλίδας

$\Sigma\tau = 0$ (Κ) $\Rightarrow$ $T_2 R_K - T_{GT} R_K = 0$

$\Rightarrow T_2 = T_{GT}$

$\Rightarrow T_{GT} = 20\,N$

$\Sigma F_y = 0 \Rightarrow$

$F + W_x - T_2 - T_{GT} = 0 \Rightarrow$

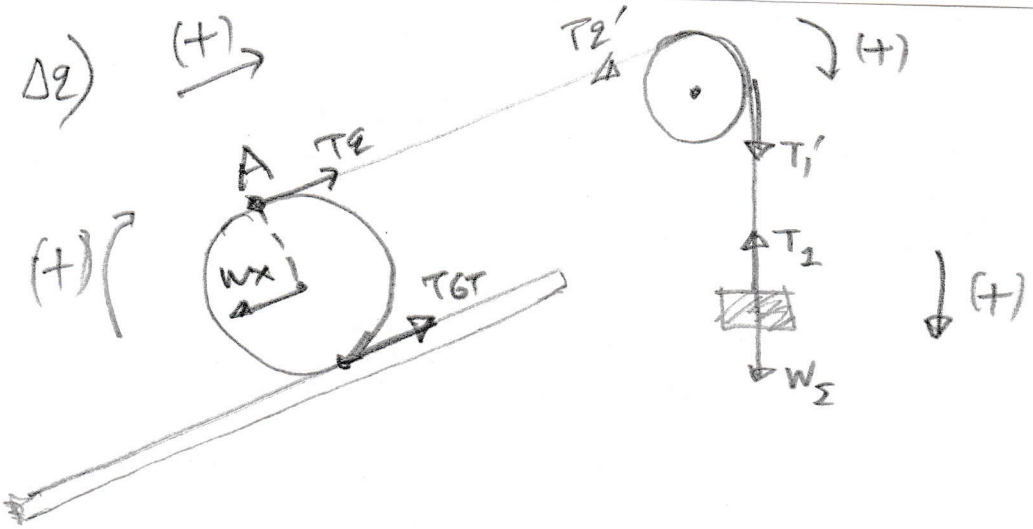
$F + M_K g \cdot \eta\mu\phi = T_2 + T_{GT}$

$F + 20 \cdot 0{,}5 = 40$

$F = 40 - 10$

$F = 30\,N$

Δ2)



(Σ) $\Sigma F_y = M_\Sigma \cdot a_\Sigma \Rightarrow$

$M_\Sigma \cdot g - T_1 = M_\Sigma a_{\text{αμω}_\Sigma} \Rightarrow$

$20 - T_1 = 2 \cdot a_\Sigma \Rightarrow$

$\boxed{T_1 = 20 - 2\,a_\Sigma}$ (1)

Τροχαλία

$\Sigma \tau = 0 \Rightarrow$

$T_1 \cdot R_T - T_2 \cdot R_T = I_T \cdot \alpha_{\text{γωνT}} \Rightarrow$ (2)

$T_1 \cdot R_T - T_2 \cdot R_T = \frac{1}{2} M_T R_T^2 \cdot \frac{a_\Sigma}{R_T}$

$T_1 - T_2 = \frac{1}{2} \cdot M_T \cdot a_\Sigma \Rightarrow$

$\boxed{T_1 - T_2 = a_\Sigma}$ (3)

Το νήμα δεν γλιστράει

$a_{\text{γραμ}} = a_{\text{γωνΤροχ}} \cdot R_T \Rightarrow$

$a_\Sigma = a_{\text{γωνT}} \cdot R_T$ (2)

(1)

$\Rightarrow 20 - 2a_\Sigma - T_2 = a_\Sigma$

$\Rightarrow 20 - T_2 = 3 \cdot a_\Sigma$

$\Rightarrow \boxed{T_2 = 20 - 3 \cdot a_\Sigma}$ (4)

Κύλινδρος

Κύλιση χωρίς ολίσθηση

$a_{\text{αμ}_K} = a_{\text{γωνK}} \cdot R_K$ (5)

Μεταφορική

$\Sigma F_x = M_K \cdot a_{\text{αμ}_K} \Rightarrow$

$T_2 + T_{GT} - W_x = M_K \cdot a_{\text{αμ}_K} \Rightarrow$

$T_2 + T_{GT} - M_K g \cdot \text{ημφ} = M_K \, a_{\text{αμ}_K} \Rightarrow$

$T_2 + T_{GT} - 10 = 2 \cdot a_{\text{αμ}_K}$ (6)

Στροφική

$\Sigma\tau = I_K \alpha_{\gamma\omega\nu K}$ (5) $\Rightarrow$

$T_2 \cdot R_K - T_{GT} \cdot R_K = \frac{1}{2} M_K R_K^2 \cdot \frac{a_{\omega\nu K}}{R_K}$

$T_2 - T_{GT} = \frac{1}{2} \cdot 2 \cdot a_{\omega\nu K}$

$\boxed{T_2 - T_{GT} = a_{\omega\nu K}}$ (7)

Οπως $U_A = U_\Sigma$
(στερεωτικο νημα)

και $U_A = 2 U_{\omega\nu K}$
(σημειο σημειο τροχου που κυλιεται)

Αρα $\frac{\Delta U_A}{\Delta t} = 2 \frac{\Delta U_{\omega\nu K}}{\Delta t}$

$a_A = 2 \cdot a_{\omega\nu K}$

$\Rightarrow \boxed{a_\Sigma = 2 \cdot a_{\omega\nu K}}$ (8)

(6)+(7) $\Rightarrow$ $2T_2 + T_{GT} - T_{GT} - 10 = 3 a_{\omega\nu K}$

$\Rightarrow 2T_2 - 10 = 3 \cdot a_{\omega\nu K}$

(4) $\Rightarrow 2(20 - 3a_\Sigma) - 10 = 3 \cdot a_{\omega\nu K}$

$\Rightarrow 40 - 6a_\Sigma - 10 = 3 \cdot a_{\omega\nu K}$

$\Rightarrow 30 - 6a_\Sigma = 3 \cdot a_{\omega\nu K}$

(8) $\Rightarrow 30 - 12 a_{\omega\nu K} = 3 \cdot a_{\omega\nu K}$

$\Rightarrow 30 = 15 \cdot a_{\omega\nu K}$

$\Rightarrow a_{\omega\nu K} = \frac{30}{15} \Rightarrow \boxed{a_{\omega\nu K} = 2 m/s^2}$

(8) $\Rightarrow \boxed{a_\Sigma = 4 m/s^2}$

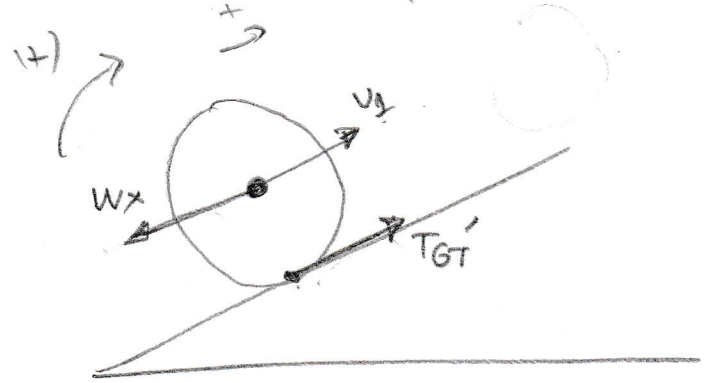
Δ3) Την $t_1 = 0,5\,sec$

$U_1 = a_{\alpha\nu\omega} t_1$

$U_1 = 2 \cdot 0,5$

$\underline{U_1 = 1\,m/s}$

Μόλις το πήρα $T_2 = 0$



Έχουμε $a_{\alpha\nu\acute{\iota}\kappa}$ (Νέα επιβράδυνση)

$\Sigma F_x = M_K a_{\alpha\nu\acute{\iota}\kappa} \Rightarrow$

$T_{GT'} - M_g \cdot \eta\mu\phi = M_K a_{\alpha\nu\acute{\iota}\kappa} \Rightarrow$

$T_{GT'} - 10 = 2\, a_{\alpha\nu\acute{\iota}\kappa} \Rightarrow$

$\boxed{T_{GT'} = 2\, a_{\alpha\nu\acute{\iota}\kappa} + 10}$

Μη ολίσθηση

$a_{\alpha\nu\acute{\iota}\kappa} = a_{\gamma\acute{\omega}\nu} R_K$

$\Sigma \tau = I_K a_{\gamma\acute{\omega}\nu} \Rightarrow$

$-T_{GT'} R_K = \frac{1}{2} M_K R_K^2 \frac{a_{\alpha\nu\acute{\iota}\kappa}}{R_K} \Rightarrow$

$-T_{GT'} = \frac{1}{2} R_K a_{\alpha\nu\acute{\iota}\kappa}$

$\boxed{T_{GT'} = -a_{\alpha\nu\acute{\iota}\kappa}}$

$2a_{\alpha\nu\acute{\iota}\kappa} + 10 = -a_{\alpha\nu\acute{\iota}\kappa} \Rightarrow$

$3a_{\alpha\nu\acute{\iota}\kappa} = -10$

$a_{\alpha\nu\acute{\iota}\kappa} = -\frac{10}{3}\,m/s^2$

$(-) \Rightarrow$ επιβράδυνση

Άρα $U_2 = U_1 - |a_{\alpha\nu\acute{\iota}\kappa}| \Delta t_2$

$0 = 1 - \frac{10}{3} \Delta t_2 \Rightarrow$

$\boxed{\Delta t_2 = 0,3\,sec}$ διάρκεια επιβράδυνσης

$t_2 = t_1 + \Delta t_2 = \boxed{0,8\,sec}$

$\Delta x_{ολ} = \Delta x_1 + \Delta x_2$

$\Delta x_1 = \frac{1}{2} \cdot a_{απ_K} \cdot t_1^2 \Rightarrow \frac{1}{2} \cdot 2 \cdot 0,5^2 = 0,25m$

$\Delta x_2 = U_1 \cdot \Delta t_2 - \frac{1}{2} |a_{απ_K}| \cdot \Delta t_2^2 = 1 \cdot 0,3 - \frac{1}{2} \cdot \frac{5}{3} \cdot 0,3^2$

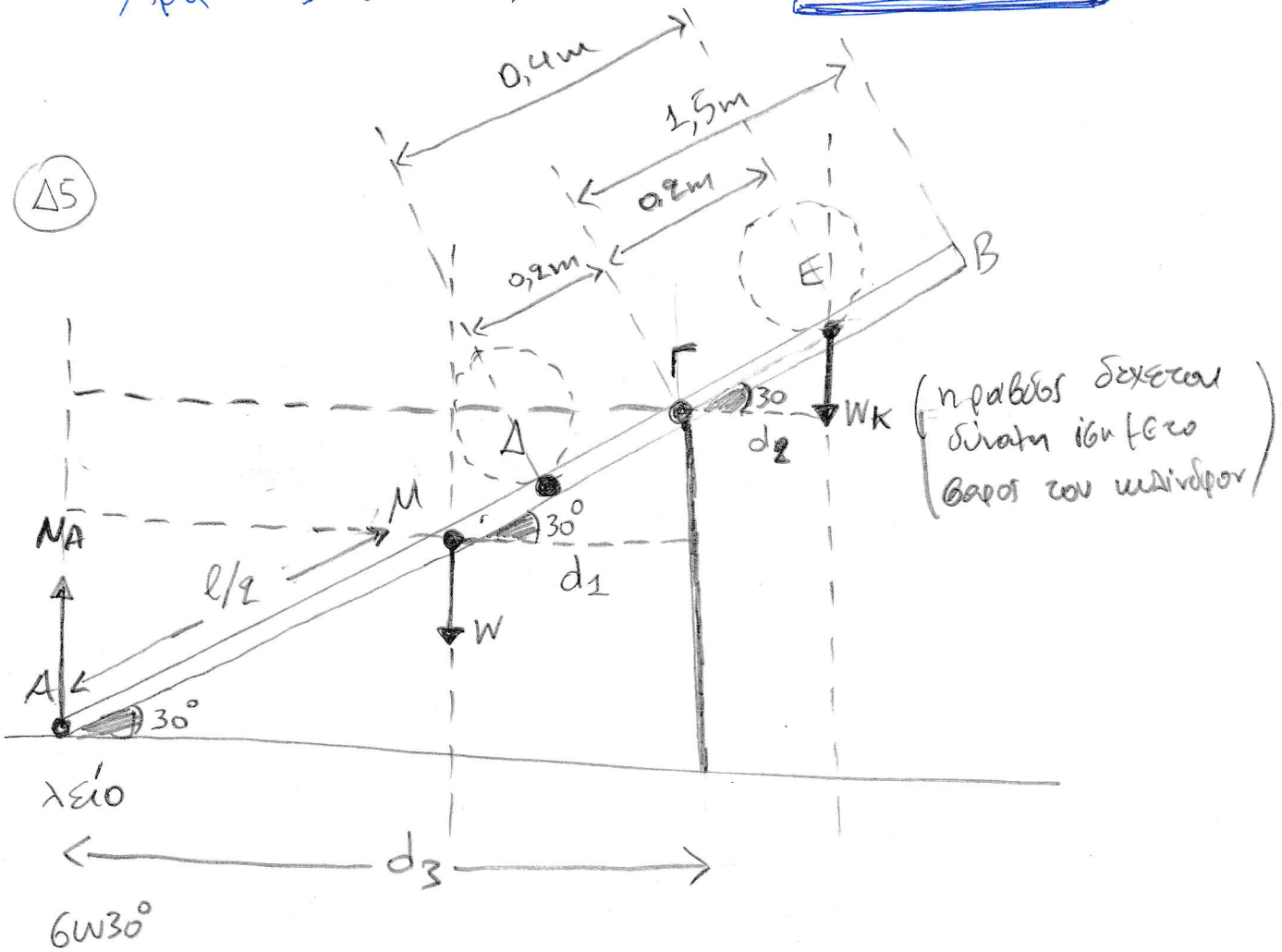
$\Delta x_2 = 0,3 - \frac{5}{0,3} \cdot 0,09$

$\Delta x_2 = 0,3 - 0,15$

$\Delta x_2 = 0,15m$

Άρα $\Delta x_{ολ} = 0,25 + 0,15 \Rightarrow \boxed{\Delta x_{ολ} = 0,4m}$

Δ5



$\delta\upsilon 30^\circ$

$\delta\upsilon 30^\circ = \frac{d_3}{A\Gamma} \Rightarrow d_3 = A\Gamma \, \delta\upsilon 30^\circ = (l - B\Gamma)\frac{\sqrt{3}}{2} = (4 - 1,5)\cdot\frac{\sqrt{3}}{2} \Rightarrow \underline{d_3 = 2,5\frac{\sqrt{3}}{2}\ m}$

$$\sigma\upsilon\nu 30^\circ = \frac{d_2}{\Gamma E} \Rightarrow d_2 = \Gamma E \, \sigma\upsilon\nu 30^\circ = 0{,}2 \frac{\sqrt{3}}{2} \Rightarrow \underline{d_2 = 0{,}1\sqrt{3}\,m}$$

$$\Gamma E = \Delta x_{o\lambda} - \Gamma\Delta = 0{,}4 - 0{,}2 = 0{,}2\,m$$

$$\sigma\upsilon\nu 30^\circ = \frac{d_1}{\Gamma M} \Rightarrow d_1 = \Gamma M \cdot \sigma\upsilon\nu 30^\circ = 0{,}5\frac{\sqrt{3}}{2} \Rightarrow \underline{d_1 = 0{,}25\sqrt{3}\,m}$$

$$\begin{aligned}\Gamma M &= A\Gamma - AM \\ &= (\ell - B\Gamma) - \frac{\ell}{2} \\ &= (4 - 1{,}5) - 2 \\ &= 2{,}5 - 2 \\ &= 0{,}5\,m\end{aligned}$$

Για να μην ανατρέπεται πρέπει όσην ώρα ασκείται δύναμη $N_A \neq 0$.

$$\Sigma \vec{\tau}_{(\Gamma)} = 0 \Rightarrow$$

$$\vec{\tau}_{N_A} + \vec{\tau}_W + \vec{\tau}_{W_K} = 0 \Rightarrow$$

$$-N_A d_3 + W \cdot d_1 - W_K \cdot d_2 = 0 \Rightarrow$$

$$N_A \cdot d_3 = Mgd_1 - M_K g \cdot d_2 \Rightarrow$$

$$N_A \cdot \frac{2{,}5}{2}\sqrt{3} = 20 \cdot 0{,}25\sqrt{3} - 20 \cdot 0{,}1\sqrt{3} \Rightarrow$$

$$N_A \frac{2{,}5}{2} = 20\frac{1}{4} - 2$$

$$\frac{2{,}5}{2} N_A = 5 - 2 = 3$$

$$N_A = \frac{6}{2{,}5} \Rightarrow \boxed{N_A = 2{,}4\,N} \neq 0$$

Άρα ΔΕΝ ανατρέπεται.